EE 435

Lecture 29

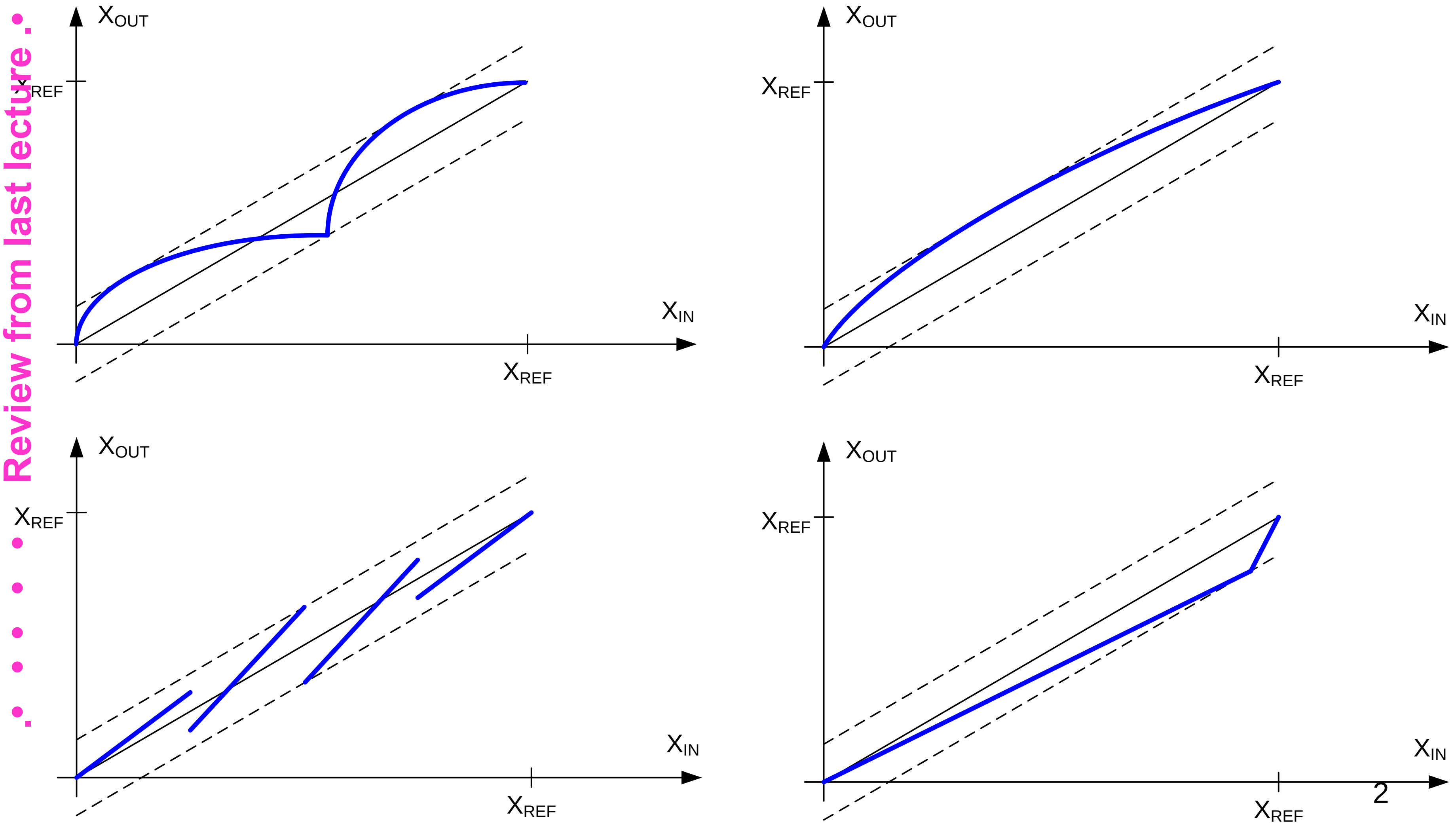
Data Converters

- Spectral Performance

INL Often Not a Good Measure of Linearity

Review from last lecture

Four identical INL with dramatically different linearity

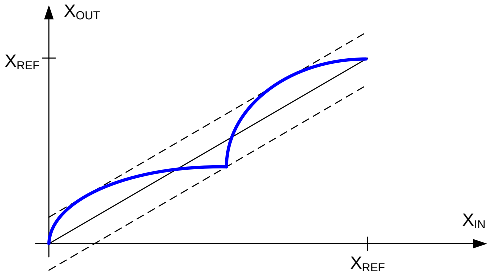


Linearity Issues

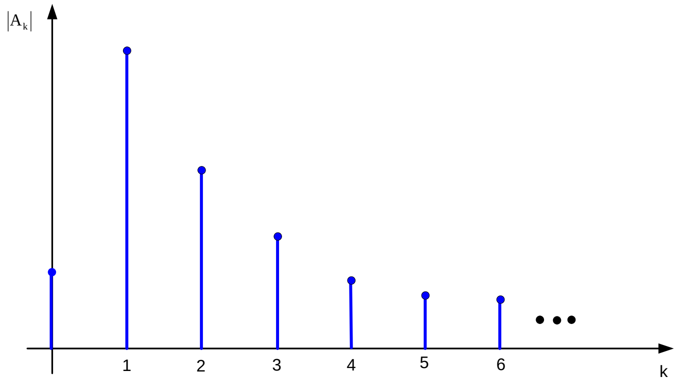
- INL is often not adequate for predicting the linearity performance of a data converter
- Distortion (or lack thereof) is of major concern in many applications
- Distortion is generally characterized in terms of the harmonics that may appear in a waveform

Two Popular Methods of Linearity Characterization

- Integral and Differential *Nonlinearity* *(metrics: INL,* DNL)



- Spectral Characterization (Based upon spectral harmonics of sinusoidal signals   metrics: THD, SFDR, SDR SNR)



Review from last lecture

Spectral Characterization

Assume x(t) is periodic with period T (T=1/f) and band-limited to Mf

Fourier Series Representation: $x(t)=\sum_{i=1}^{M} A_k \sin(k\omega t+\theta_k)$ 2M parameters (A_k, θ_k)

If x(t) is uniformly sampled 2M times with sampling interval T_s where $2MT_s=T$

Time domain sequence: $\vec{x} = < x(T_S), x(2T_S), \ldots . x(2MT_S) >$ 2M parameters

IDFT

Termed DFT

DFT

2M parameters

Denoted as frequency domain sequence: $\vec{X} = < X_1, X_2,,,, X_M >$

$$X_k = A_k e^{j\theta_k}$$

- 2M time domain samples spaced as specified completely characterizes x(t) for all t
- Frequency domain sequence $\vec{X}$ completely characterizes x(t) for all t

Distortion Analysis

Review from last lecture

Total Harmonic Distortion, THD

$$\mathrm{THD} = \frac{\text{RMS voltage in harmonics}}{\text{RMS voltage of fundamental}}$$

$$\mathrm{THD} = \frac{\sqrt{\left(\frac{A_2}{\sqrt{2}}\right)^2 + \left(\frac{A_3}{\sqrt{2}}\right)^2 + \left(\frac{A_4}{\sqrt{2}}\right)^2 + \ldots}}{\frac{A_1}{\sqrt{2}}}$$

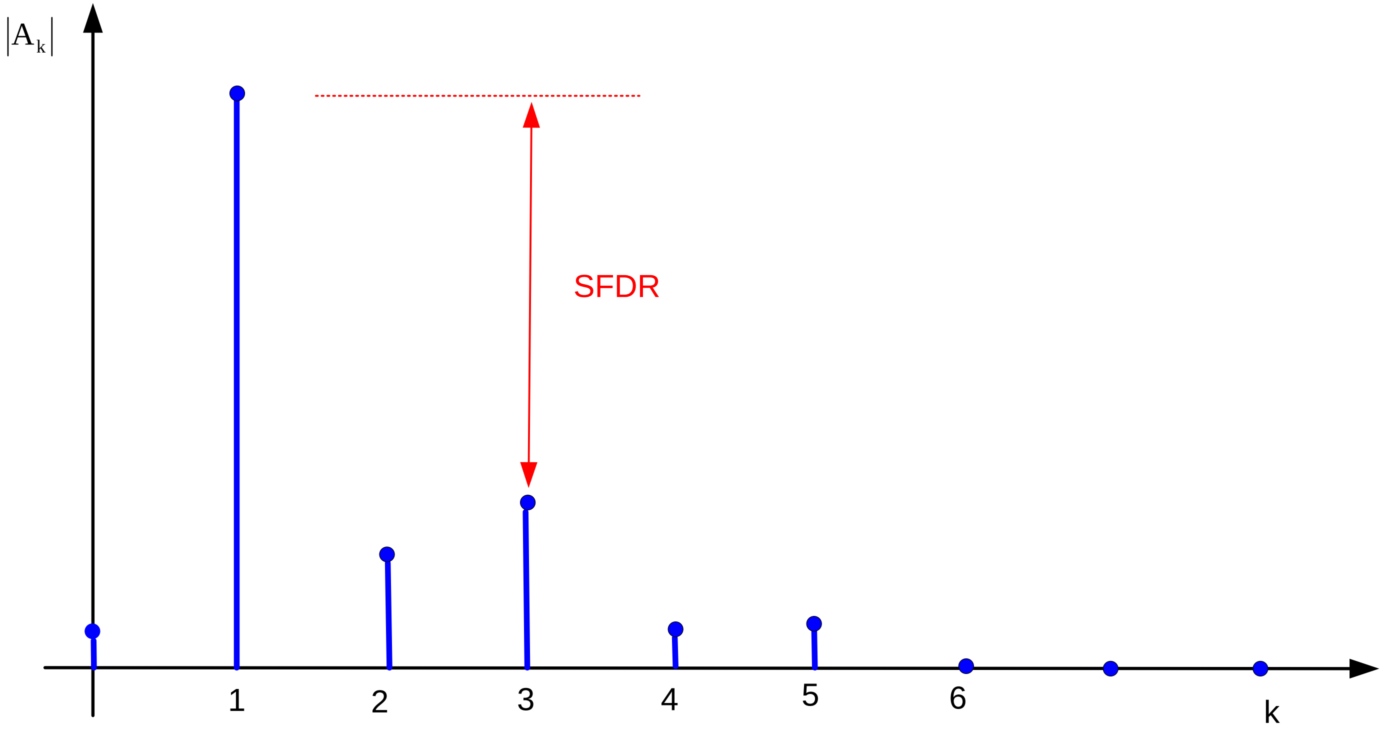
$$\mathrm{THD} = \frac{\sqrt{\sum_{k=2}^{\infty} A_k^2}}{A_1}$$

Distortion Analysis

Review from last lecture

Spurious Free Dynamic Range, SFDR

The SFDR is the difference between the fundamental and the largest harmonic



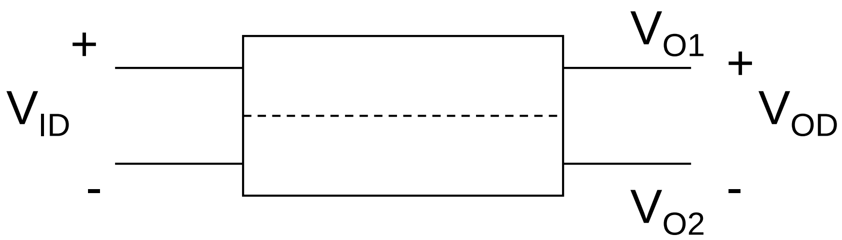
SFDR is usually determined by either the second or third harmonic

Review from last lecture

Distortion Analysis

Theorem: In a fully differential symmetric circuit, all even harmonics are absent in the differential output for symmetric differential sinusoidal excitations !

Proof:



Expanding in a Taylor's series around $V_{ID}=0$, we obtain

$$V_{O1} = x(V_{ID}) = \sum_{k=0}^{\infty} h_k V_{ID}^k \quad \text{and} \quad V_{O2} = x(-V_{ID}) = \sum_{k=0}^{\infty} h_k (-V_{ID})^k$$

Assume $V_{ID}=K\sin(\omega t)$ W.L.O.G. assume K=1

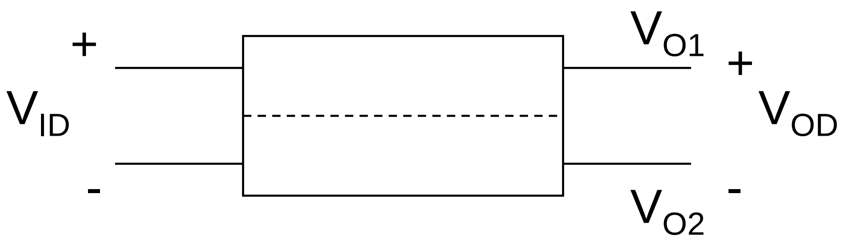
$$V_{O1} = \sum_{k=0}^{\infty} h_k [\sin(\omega t)]^k \qquad V_{O2} = \sum_{k=0}^{\infty} h_k [-\sin(\omega t)]^k$$

$$V_{OD} = V_{O1} - V_{O2} = \sum_{k=0}^{\infty} h_k \left([\sin(\omega t)]^k - [-\sin(\omega t)]^k \right) = \sum_{k=0}^{\infty} h_k \left([\sin(\omega t)]^k - (-1)^k [\sin(\omega t)]^k \right)$$

Observe the even-ordered powers and hence even harmonics are absent in this last sum

Distortion Analysis

Theorem: In a fully differential symmetric circuit, all even harmonics are absent in the differential output for symmetric differential excitations !



Proof:

Recall:

$$\sin^n(x) = \begin{cases} \sum_{k=0}^{\frac{n-1}{2}} h_k \sin\left(\left(n-2k\right)x\right) & \textit{for n odd} \\ \sum_{k=0}^{\frac{n-2}{2}} g_k \sin\left(\left(n-2k\right)x+\theta_k\right) & \textit{for n even} \end{cases}$$

where h_k, g_k, and θ_k are constants

That is, odd powers of $\sin^n(x)$ have only odd harmonics present and even powers have only even harmonics present

Distortion Analysis

How are spectral components determined?

By integral

$$A_k = \frac{1}{\omega T}\left(\int_{t_1}^{t_1+T} x(t)e^{-jk\omega t}dt + \int_{t_1}^{t_1+T} x(t)e^{jk\omega t}dt \right)$$

or

$$a_k = \frac{2}{\omega T}\int_{t_1}^{t_1+T} x(t)\sin(kt\omega)dt \qquad b_k = \frac{2}{\omega T}\int_{t_1}^{t_1+T} x(t)\cos(kt\omega)dt$$

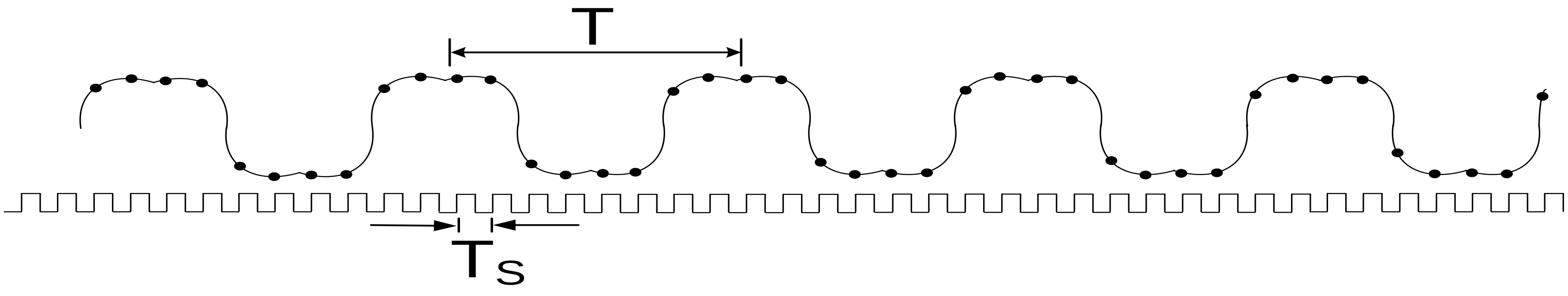
Integral is very time consuming, particularly if large number of components are required

By DFT (with some restrictions that will be discussed)

By FFT (special computational method for obtaining DFT)

Distortion Analysis

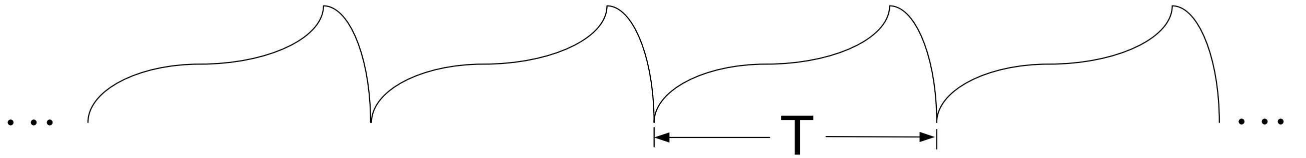
How are spectral components determined?



Consider sampling x(t) at uniformly spaced points in time T_S seconds apart

This gives a sequence of samples $\left\langle x\left(kT_s\right)\right\rangle_{k=1}^{N}$

Distortion Analysis



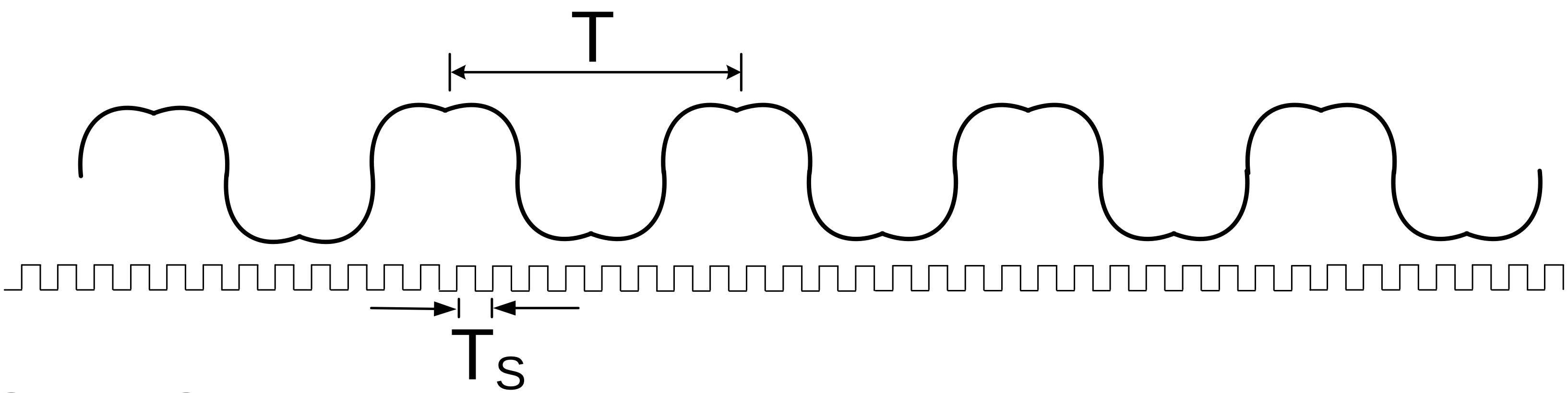
Consider a function x(t) that is periodic with period T

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin(k\omega t + \theta_k) \qquad \omega = 2\pi f = \frac{2\pi}{T}$$

Band-limited Periodic Functions

Definition: A periodic function of frequency f is band limited to a frequency f_{max} if A_k=0 for all $k > \frac{f_{max}}{f}$

Distortion Analysis



NOTATION:

- T: Period of Excitation
- T_S: Sampling Period
- N_P: Number of periods over which samples are taken
- N: Total number of samples

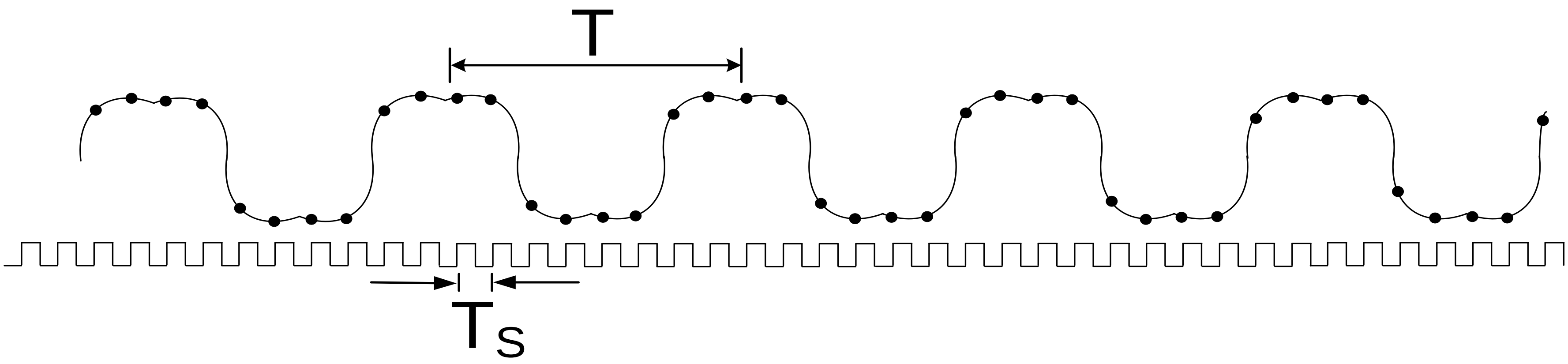
$$N_P = \frac{NT_S}{T}$$

Note: N_P is not an integer unless a specific relationship exists between N, T_S and T

$$h = \mathrm{Int}\left(\left[\frac{N}{2}-1\right]\frac{1}{N_P}\right)$$

Note: The function Int(x) is the integer part of x

Distortion Analysis



Observation : If a band-limited periodic signal is sampled over an integral number of periods at a rate that exceeds the Nyquist rate, then the Fourier Series coefficients can be directly obtained from the sampled sequence.

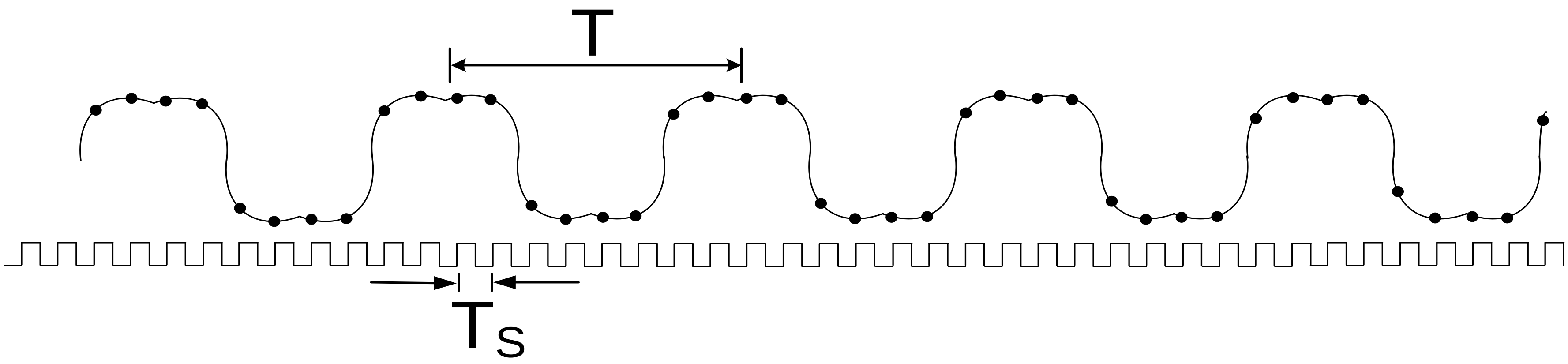
$$x(t) = A_0 + \sum_{k=1}^{N_X} A_k \sin(k\omega t + \theta_k) \qquad \omega = 2\pi \bullet f_{sig}$$

Band-limited to N_x implications $\quad A_{Nx} \neq 0 \quad A_k = 0$ for all $k > N_x$

Number of unknowns: $2N_x+1$

$f_{NYQ} = 2N_x f_{sig}$ If sampled at Nyquist rate for 1 period of signal will have $2N_x$ samples

Distortion Analysis

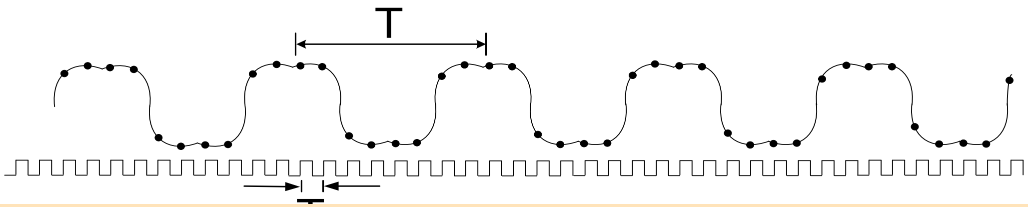


THEOREM (conceptual) : If a band-limited periodic signal is sampled N times at a rate that exceeds the Nyquist rate, then the Fourier Series coefficients can be directly obtained from the DFT of a sampled sequence.

$$\left\langle x(kT_S)\right\rangle_{k=0}^{N-1} \Longleftrightarrow \left\langle X(k)\right\rangle_{k=0}^{N-1}$$

Because there is sufficient information in the sample sequence to obtain the Fourier Series coefficients

Distortion Analysis



$$x(t) = A_0 + \sum_{k=1}^{h-1} A_k \sin(k\omega t + \theta_k)$$

THEOREM: Consider N samples of a periodic signal with period T=1/f and sampling period $T_S=1/f_S$. If N_P is an integer, x(t) is band limited to f_{MAX}, and $f_s>2f_{max}$, then

$$|A_m| = \frac{2}{N}\left|X(mN_P+1)\right| \qquad 0 \le m \le h-1$$

and $X(k)=0$ for all k not defined above

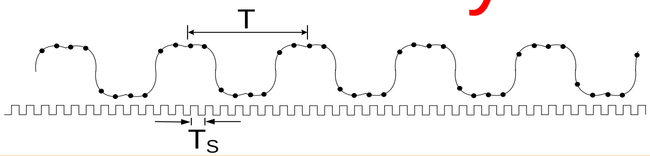
where $\langle X(k)\rangle_{k=0}^{N-1}$ is the DFT of the sequence $\langle x(kT_S)\rangle_{k=0}^{N-1}$

$\langle A_k \rangle$ are the Fourier Series Coefficients, N=number of samples, N_P is the number of periods, and $h = \text{Int}\left(\frac{f_{MAX}}{f} - \frac{1}{N_P}\right)$

Note spectral components of interest as mN_p+1

Key Theorem central to Spectral Analysis that is widely used !!! and often "abused"

Why is this a Key Theorem?



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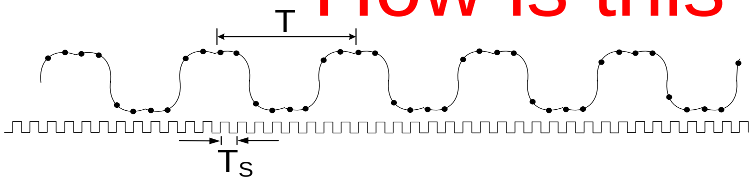
$\langle A_k\rangle$ are the Fourier Series Coefficients, N_P is the number of periods, and $h = \text{Int}\left(\frac{f_{MAX}}{f} - \frac{1}{N_P}\right)$

- DFT requires dramatically less computation time than the integrals for obtaining Fourier Series coefficients
- Can easily determine the sampling rate (often termed the Nyquist rate) to satisfy the band limited part of the theorem
- If "signal" is output of a system (e.g. ADC or DAC), f_{MAX} is independent of f

How is this theorem abused?



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$$|A_m| = \frac{2}{N}|X(mN_P+1)| \qquad 0 \le m \le h\text{-}1$$

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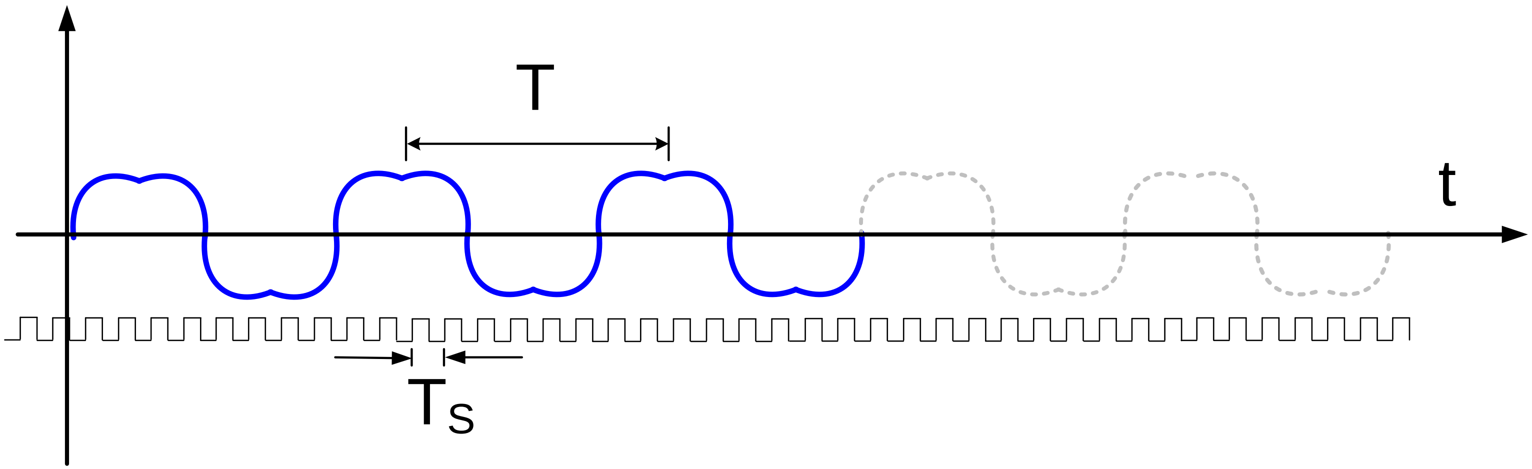
$\langle A_k \rangle$ are the Fourier Series Coefficients, N_P is the number of periods, and $h = \text{Int}\left(\frac{f_{MAX}}{f} - \frac{1}{N_P}\right)$

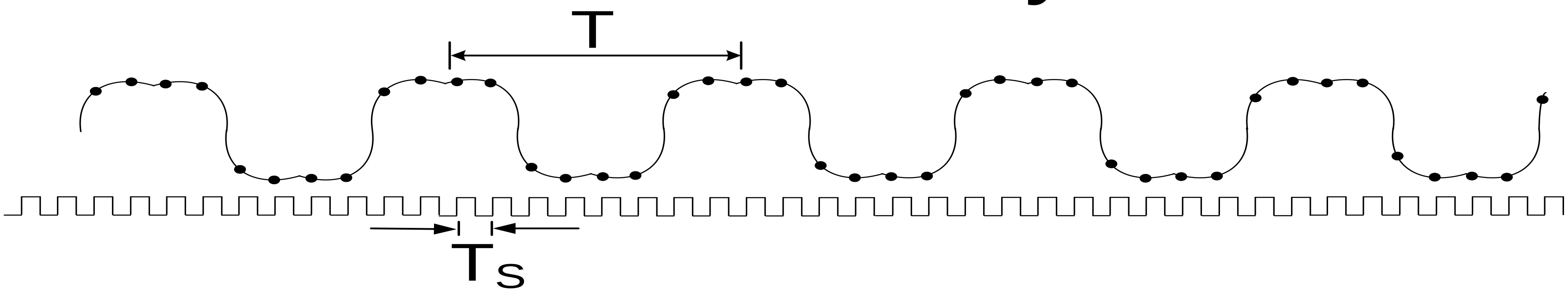
- Much evidence of engineers attempting to use the theorem when N_P is not an integer
- Challenging to have N_P an integer in practical applications
- Dramatic errors can result if there are not exactly an integer number of periods in the sampling window

3 Periods of Periodic Signal in Bold Blue

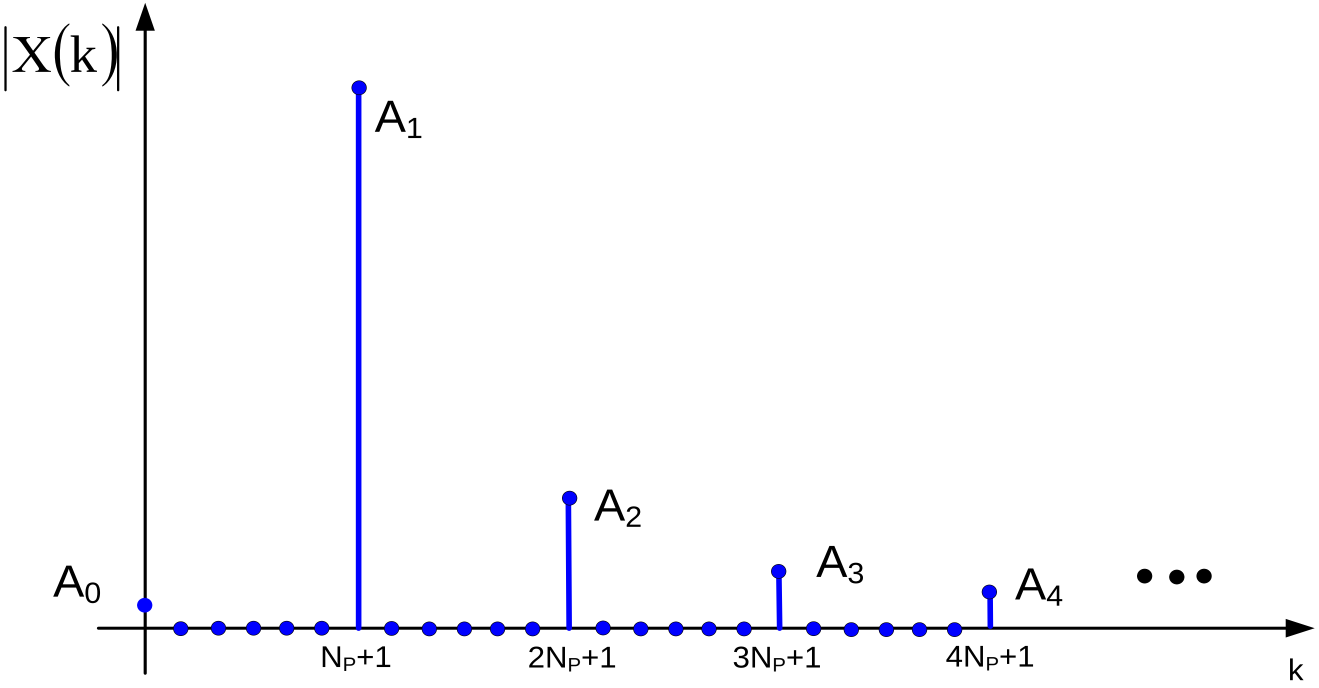


$$f_{SAMP} = f_{SIG}\frac{N}{N_P}$$

Distortion Analysis



If the hypothesis of the theorem are satisfied, we thus have



Distortion Analysis

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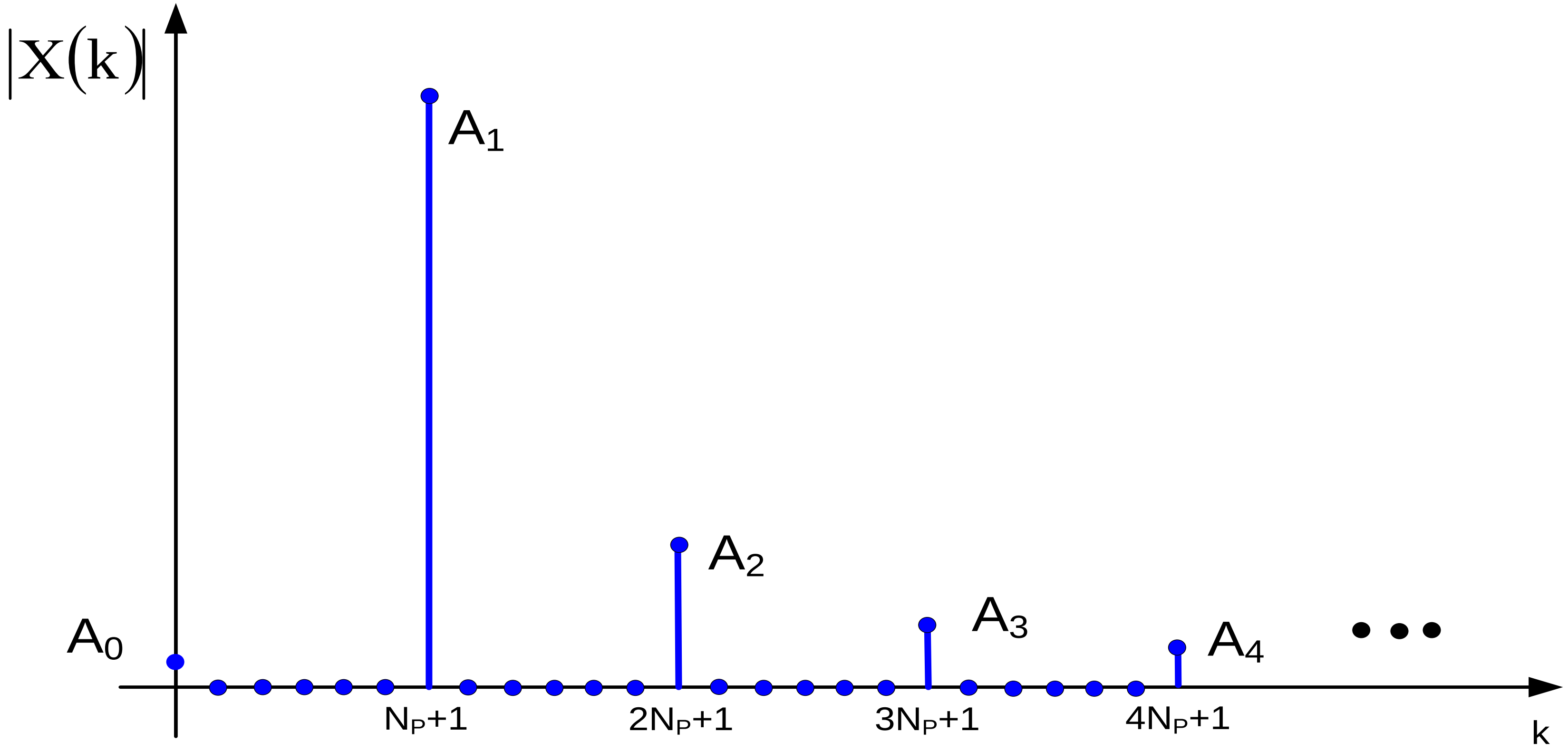

FFT is a computationally efficient way of calculating the DFT, particularly when N is a power of 2

DFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods

2. The input signal is band limited to f_{MAX}

Some notation and understanding related to Fourier Series, Discrete Fourier Series, Discrete Fourier Transform, Nyquist Rate, and Nyquist Frequency may be inconsistent from source to source, confusing, and not always correctly presented in all forums

From Wikipedia – March 30 2018

Discrete Fourier series

From Wikipedia, the free encyclopedia

A Fourier series is a representation of a function in terms of a summation of an infinite number of harmonically-related sinusoids with different amplitudes and phases. The amplitude and phase of a sinusoid can be combined into a single complex number, called a Fourier *coefficient*. The Fourier series is a periodic function. So it cannot represent any arbitrary function. It can represent either**:**

(a) a periodic function, or

(b) a function that is defined only over a finite-length interval; the values produced by the Fourier series outside the finite interval are irrelevant.

When the function being represented, whether finite-length or periodic, is discrete, the Fourier series coefficients are periodic, and can therefore be described by a finite set of complex numbers. That set is called a discrete Fourier transform (DFT), which is subsequently an overloaded term, because we don't know whether its (periodic) inverse transform is valid over a finite or an infinite interval. The term **discrete Fourier series (DFS)** is intended for use instead of *DFT* when the original function is periodic, defined over an infinite interval. *DFT* would then unambiguously imply only a transform whose inverse is valid over a finite interval. But we must again note that a Fourier series is a time-domain representation, not a frequency domain transform. So DFS is a potentially confusing substitute for DFT. A more technically valid description would be **DFS coefficients**.

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From Wikipedia – March 28 2023

Discrete Fourier series

1 language

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From Wikipedia, the free encyclopedia

In digital signal processing, the term **Discrete Fourier series** (DFS) is any periodic discrete-time signal comprising harmonically-related (i.e. *Fourier*) discrete real sinusoids or discrete complex exponentials, combined by a weighted summation. A specific example is the inverse discrete Fourier transform (inverse DFT).

Definition [edit]

The general form of a DFS is:

Discrete Fourier series

$$x[n] = \sum_{k} X[k] \cdot e^{i2\pi \frac{k}{N} n}, \quad n \in \mathbb{Z}, \qquad \textbf{(Eq.1)}$$

which are harmonics of a fundamental frequency $1/N$, for some positive integer N. The practical range of k, is $[0,\ N-1]$, because periodicity causes larger values to be redundant. When the $X[k]$ coefficients are derived from an N-length DFT, and a factor of $1/N$ is inserted, this becomes an inverse DFT.[1]:p.542 (eq 8.4) [2]:p.77 (eq 4.24) And in that case, just the coefficients themselves are sometimes referred to as a discrete Fourier series.[3]:p.85 (eq 15a)

Some notation and understanding related to Fourier Series, Discrete Fourier Series, Discrete Fourier Transform, Nyquist Rate, and Nyquist Frequency may be inconsistent and confusing

From Wikipedia – March 30 2018

Nyquist rate

From Wikipedia, the free encyclopedia

Not to be confused with Nyquist frequency.

This article **may be confusing or unclear to readers**. Please help us clarify the article. There might be a discussion about this on the talk page. *(January 2014) (Learn how and when to remove this template message)*

Nyquist frequency

From Wikipedia, the free encyclopedia

Not to be confused with Nyquist rate.

The **Nyquist frequency**, named after electronic engineer Harry Nyquist, is half of the sampling rate

The Nyquist frequency should not be confused with the *Nyquist rate*, which is the minimum sampling rate that satisfies the Nyquist sampling criterion for a given signal or family of signals. The Nyquist rate is twice the maximum component frequency of the function being sampled. For example, the *Nyquist rate* for the sinusoid at 0.6 f_s is 1.2 f_s, which means that at the f_s rate, it is being *undersampled*. Thus, *Nyquist rate* is a property of a continuous-time signal, whereas *Nyquist frequency* is a property of a discrete-time system.[4][5]

Some notation and understanding related to Fourier Series, Discrete Fourier Series, Discrete Fourier Transform, Nyquist Rate, and Nyquist Frequency may be inconsistent and confusing

From Wikipedia – March 28 2023

Nyquist rate

Article Talk

From Wikipedia, the free encyclopedia

Not to be confused with Nyquist frequency.

In signal processing, the **Nyquist rate**, named after Harry Nyquist, is a value (in units of samples per second[1] or hertz, Hz) equal to twice the highest frequency (bandwidth) of a given function or signal. When the function is digitized at a higher sample rate (see § Critical frequency), the resulting discrete-time sequence is said to be free of the distortion known as aliasing. Conversely, for a given sample-rate the corresponding Nyquist frequency in Hz is one-half the sample-rate. Note that the *Nyquist rate* is a property of a continuous-time signal, whereas *Nyquist frequency* is a property of a discrete-time system.

The term *Nyquist rate* is also used in a different context with units of symbols per second, which is actually the field in which Harry Nyquist was working. In that context it is an upper bound for the symbol rate across a bandwidth-limited baseband channel such as a telegraph line[2] or passband channel such as a limited radio frequency band or a frequency division multiplex channel.

DFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods
2. $N > \frac{2\, f_{max}}{f_{SIGNAL}} N_P$ (from $f_{MAX} \leq \frac{f}{2} \bullet \left[\frac{N}{N_P} \right]$)

Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
- Importance of Satisfying Hypothesis
- Windowing

Considerations for Spectral Characterization

- Tool Validation (MATLAB)
- DFT Length and NP
- Importance of Satisfying Hypothesis
- Windowing

Example

WLOG assume f_{SIG}=50Hz

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

$$f_{MAX\text{-}ACT} = 100\text{Hz}$$

Consider N_P=20 N=512

$$f_{MAX} = \frac{f_{SIG}}{2} \bullet \left[\frac{N}{N_P}\right] = \frac{50}{2} \bullet \frac{512}{20} = 640\,Hz$$

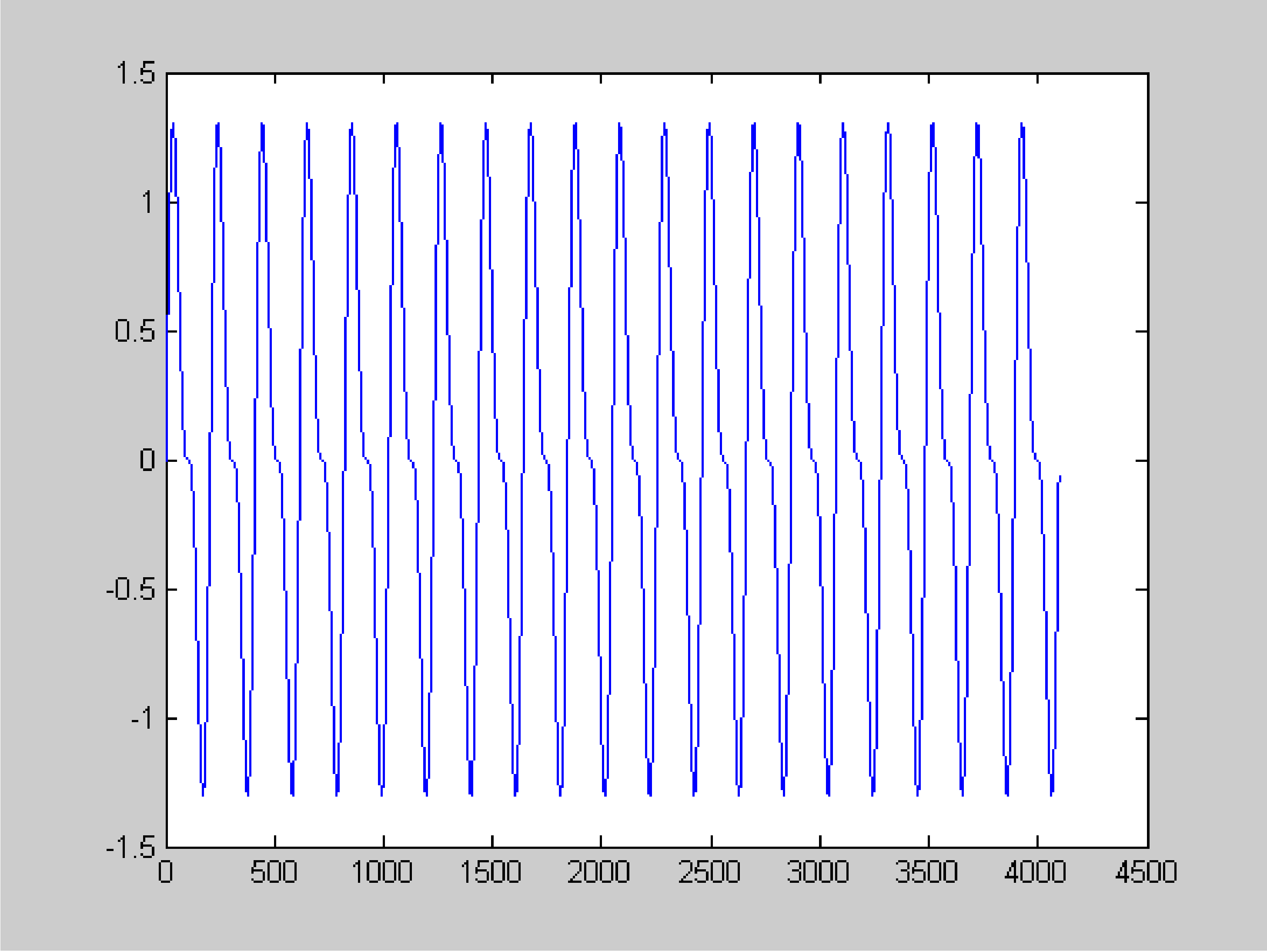
$$f_{MAX\text{-}ACT} << f_{MAX}$$

$$f_{SAMPLE} = \frac{1}{T_{SAMPLE}} = \frac{1}{\left(\frac{N_P \bullet T_{SIG}}{N}\right)} = \left[\frac{N}{N_P}\right] f_{SIG} = 2 f_{MAX} = 1280\,Hz$$

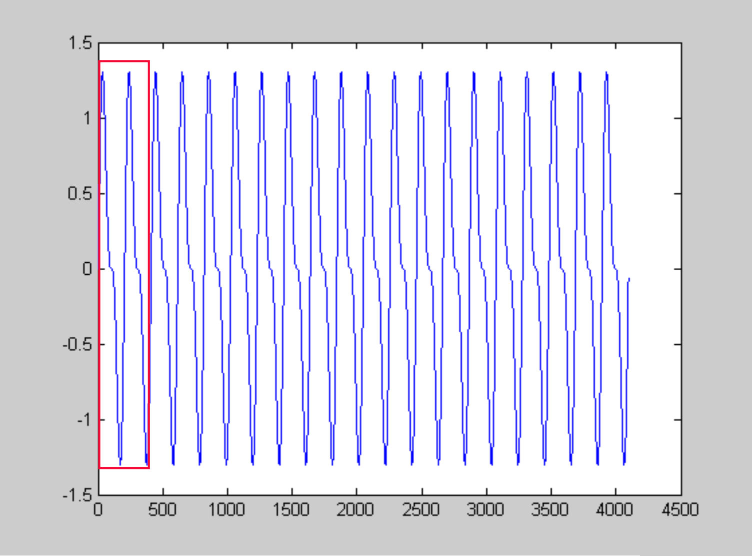
Recall $20\log_{10}(1.0)=0.0000000$

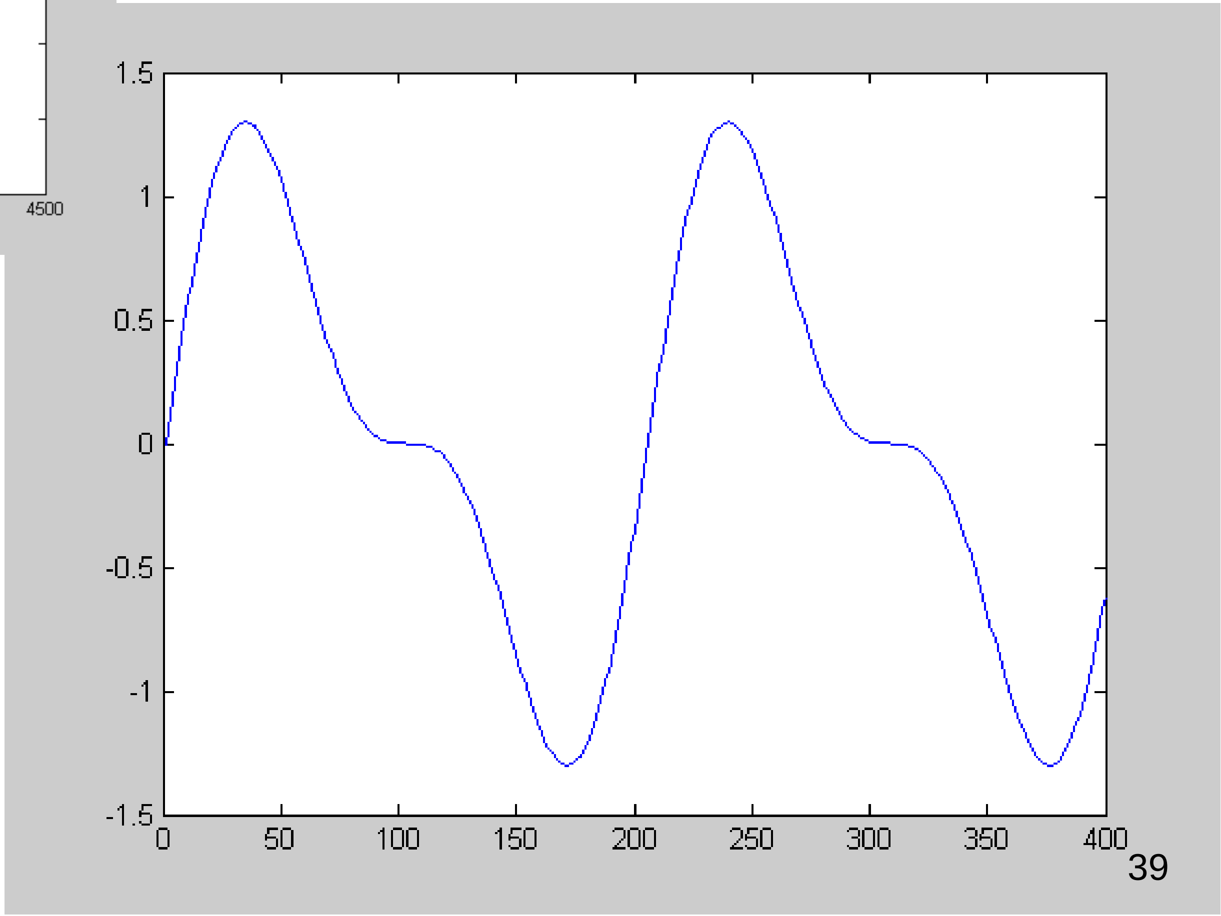
Recall $20\log_{10}(0.5)=-6.0205999$

Input Waveform



Input Waveform





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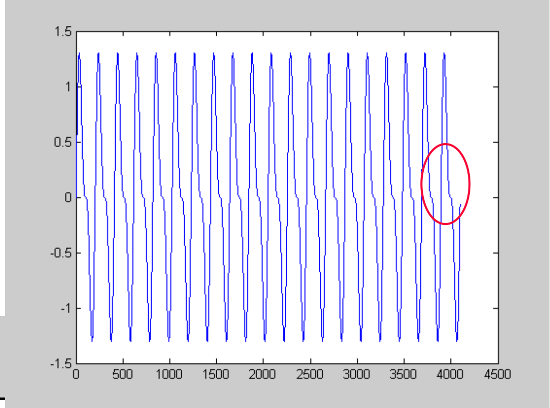
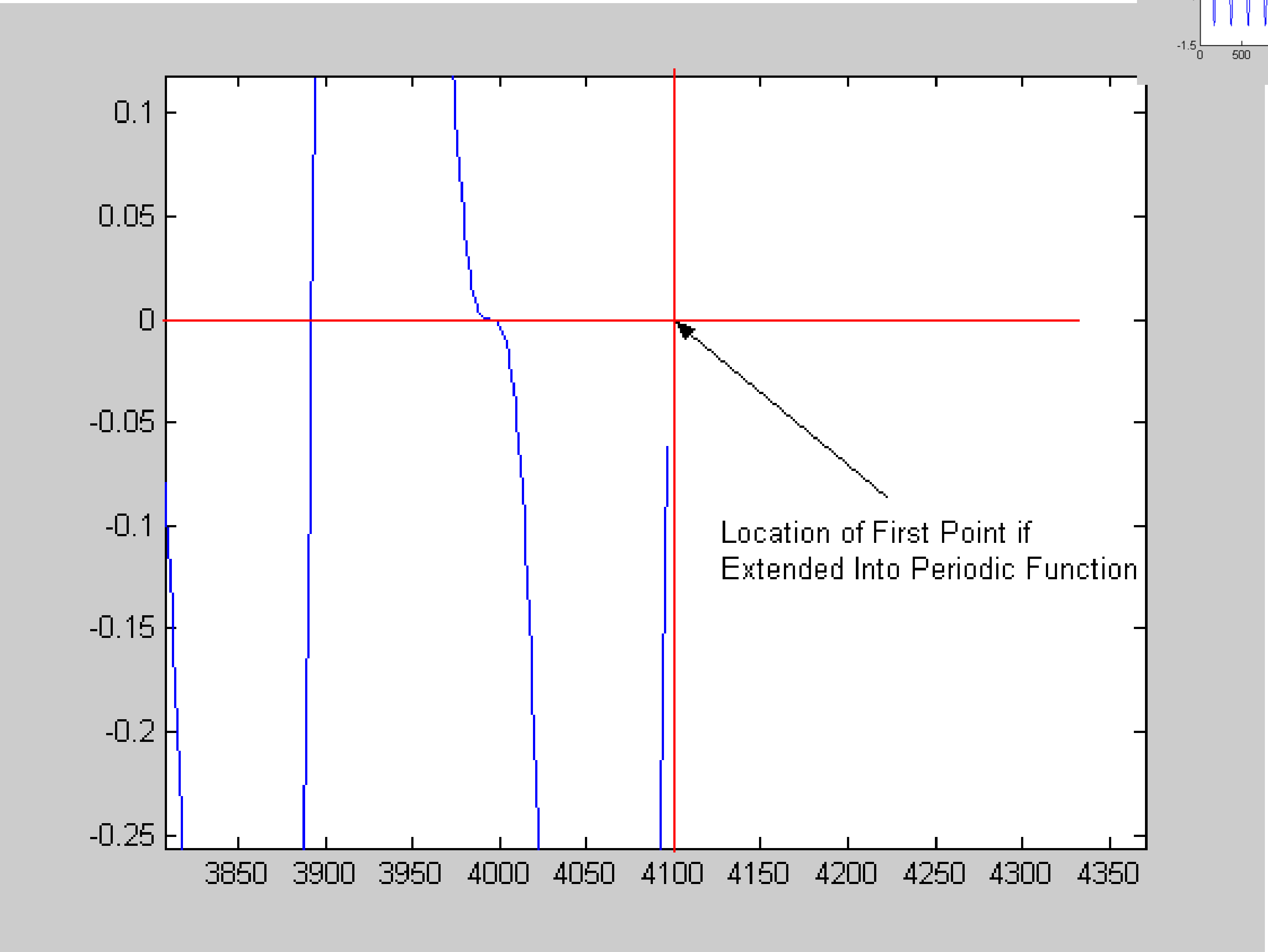
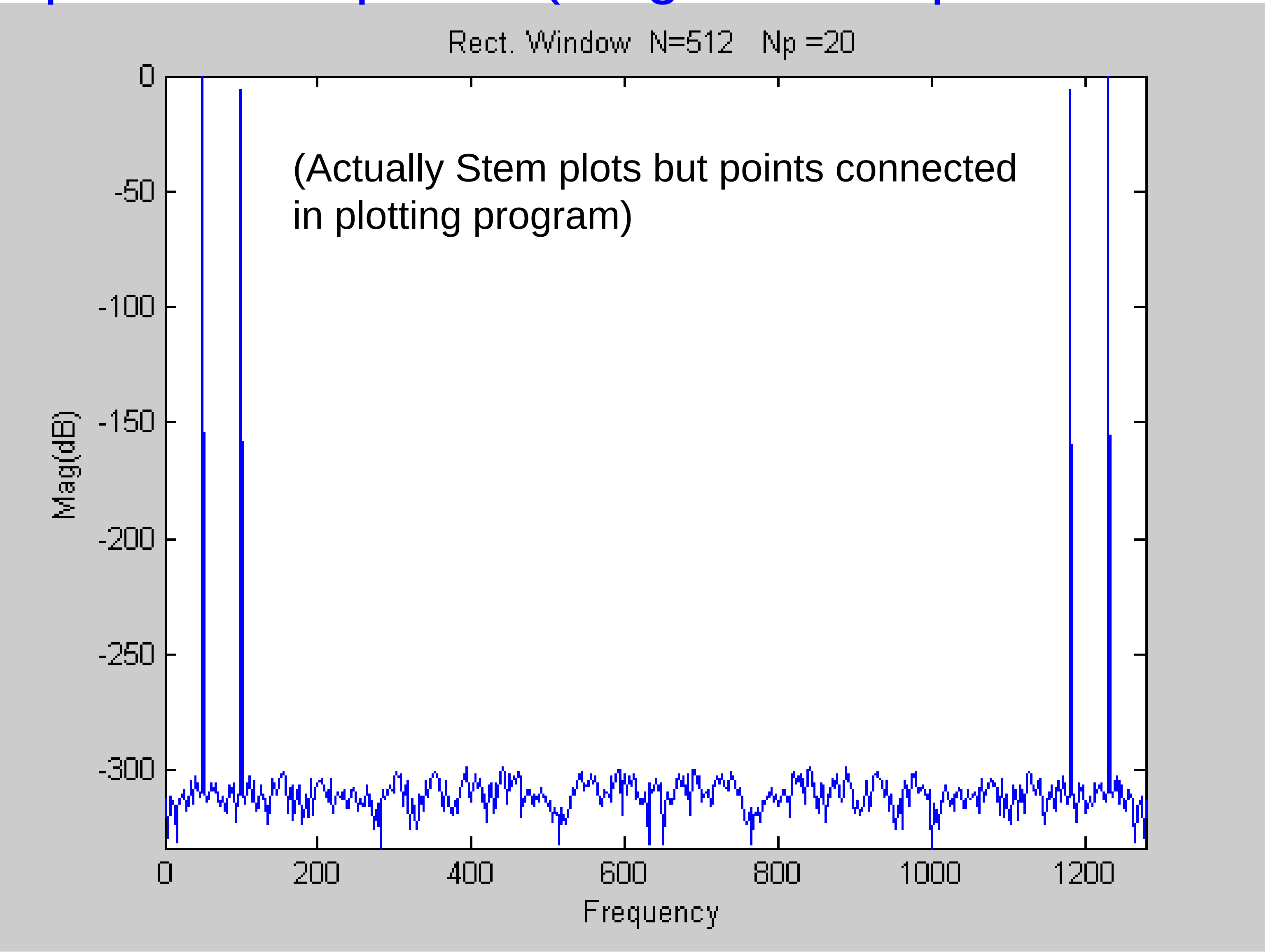




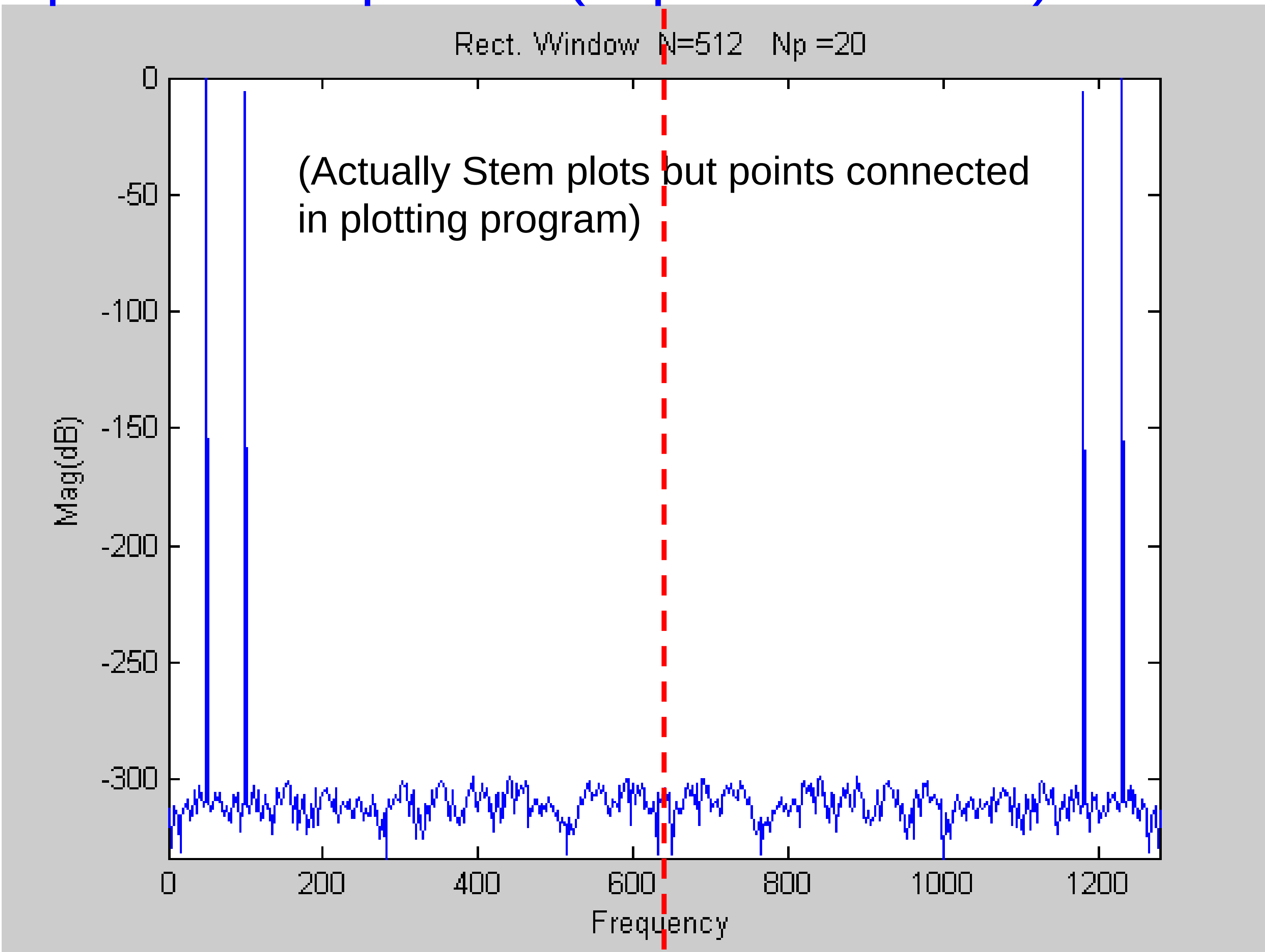
Spectral Response (magnitude expressed in dB)



(Horizontal axis is the "Index" axis but converted to frequency)

$$f_{AXIS} = f_{SIGNAL} \frac{n-1}{N_P}$$

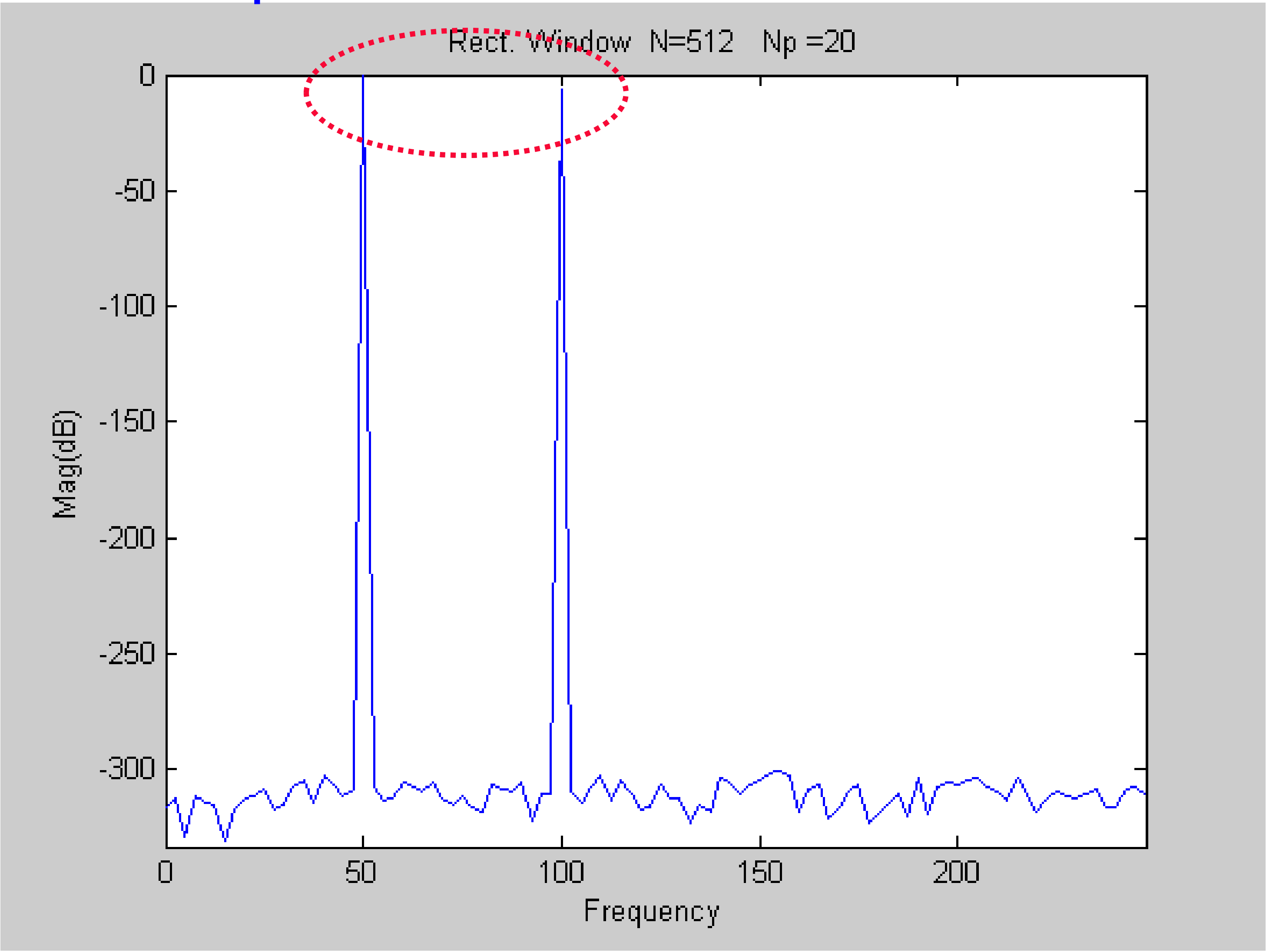
Spectral Response (expressed in dB)



Note Magnitude is Symmetric wrt $f_{SAMPLE}/2$

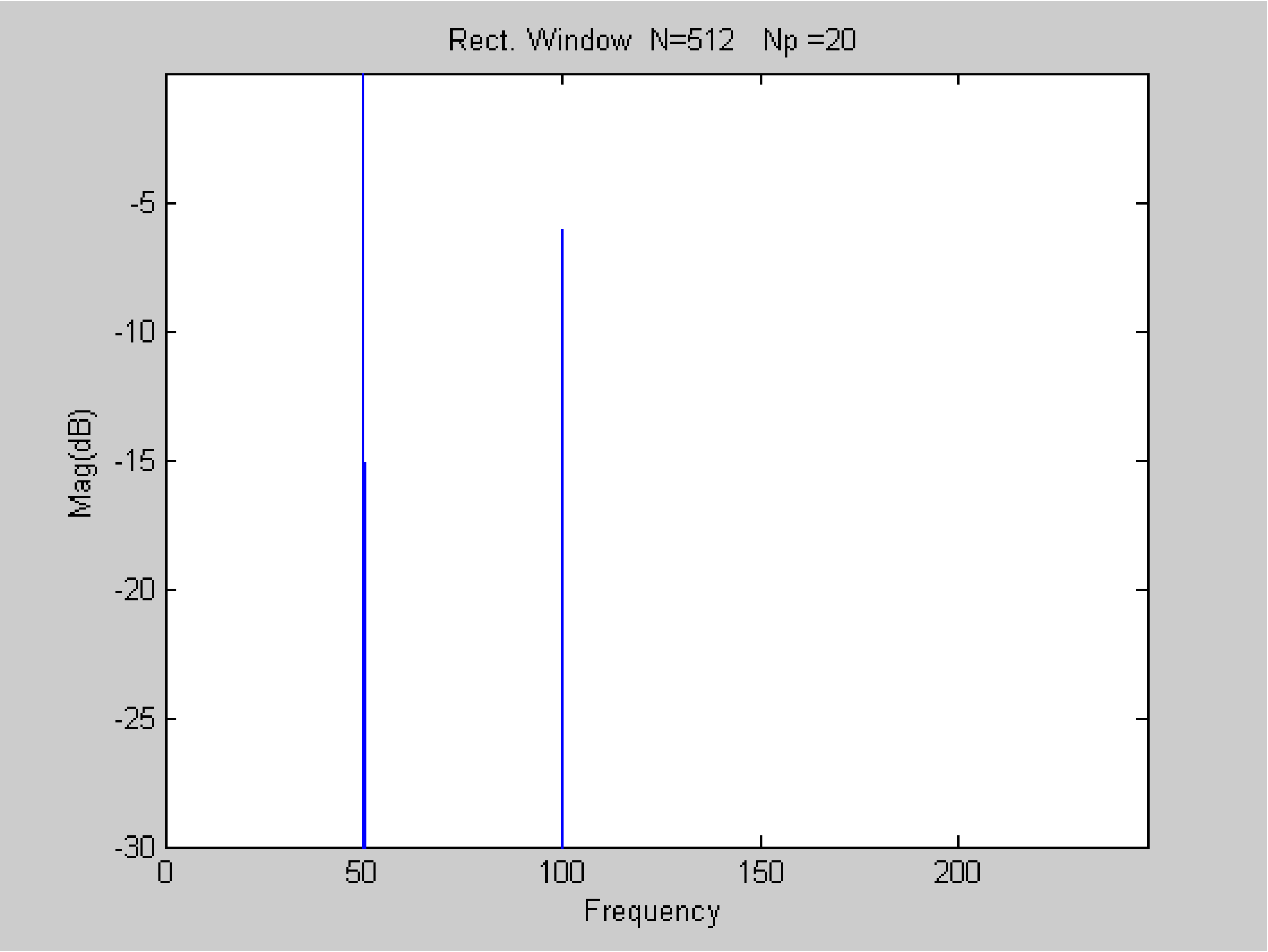
$$f_{AXIS} = f_{SIGNAL}\frac{n-1}{N_P}$$

Spectral Response



DFT Horizontal Axis Converter to Frequency : $f_{AXIS} = f_{SIGNAL} \frac{n-1}{N_P}$

Spectral Response



Fundamental will appear at position 1+Np = 21

Columns 1 through 5

-316.1458 -312.9517 -329.5203 -311.1473 -314.2615

Columns 6 through 10

-315.2584 -330.6258 -317.2896 -312.2316 -311.6335

Columns 11 through 15

-308.2339 -317.7064 -315.3135 -307.9349 -304.5641

Columns 16 through 20

-314.0088 -302.6391 -306.6650 -311.3733 -308.3689

Columns 21 through 25

-0.0000 -307.7012 -312.9902 -312.8737 -305.4320

Observe system noise floor due to both spectral limitations of signal generator and numerical limitations in FFT are below -300db

Second Harmonic at 1+2Np = 41

Columns 26 through 30

-307.8301 -309.0737 -305.8503 -312.2772 -315.7544

Columns 31 through 35

-311.9316 -316.0581 -318.3454 -306.4977 -308.6679

Columns 36 through 40

-309.9702 -305.9809 -322.1270 -310.6723 -310.3506

Columns 41 through 45

-6.0206 -309.6071 -314.1026 -307.6405 -302.9277

Columns 46 through 50

-313.0745 -304.2330 -310.8487 -317.7966 -316.3385

Third Harmonic at 1+3Np = 61

Columns 51 through 55

-307.0529 -312.7787 -312.9340 -323.2969 -314.9297

Columns 56 through 60

-318.7605 -303.5929 -305.2994 -310.6430 -306.7613

Columns 61 through 65

-304.8298 -301.4463 -301.1410 -303.1784 -317.8343

Columns 66 through 70

-308.6310 -307.0135 -321.6015 -316.6548 -309.8946

Columns 71 through 75

-306.3472 -323.0110 -319.3267 -314.7873 -310.4085

Fourth Harmonic at 1+4Np = 81

Columns 76 through 80

-319.8926 -303.3641 -319.6263 -307.6894 -305.1945

Columns 81 through 85

-306.8190 -304.8860 -303.6531 -307.2090 -309.8014

Columns 86 through 90

-313.4988 -303.4513 -310.4969 -317.9652 -312.5846

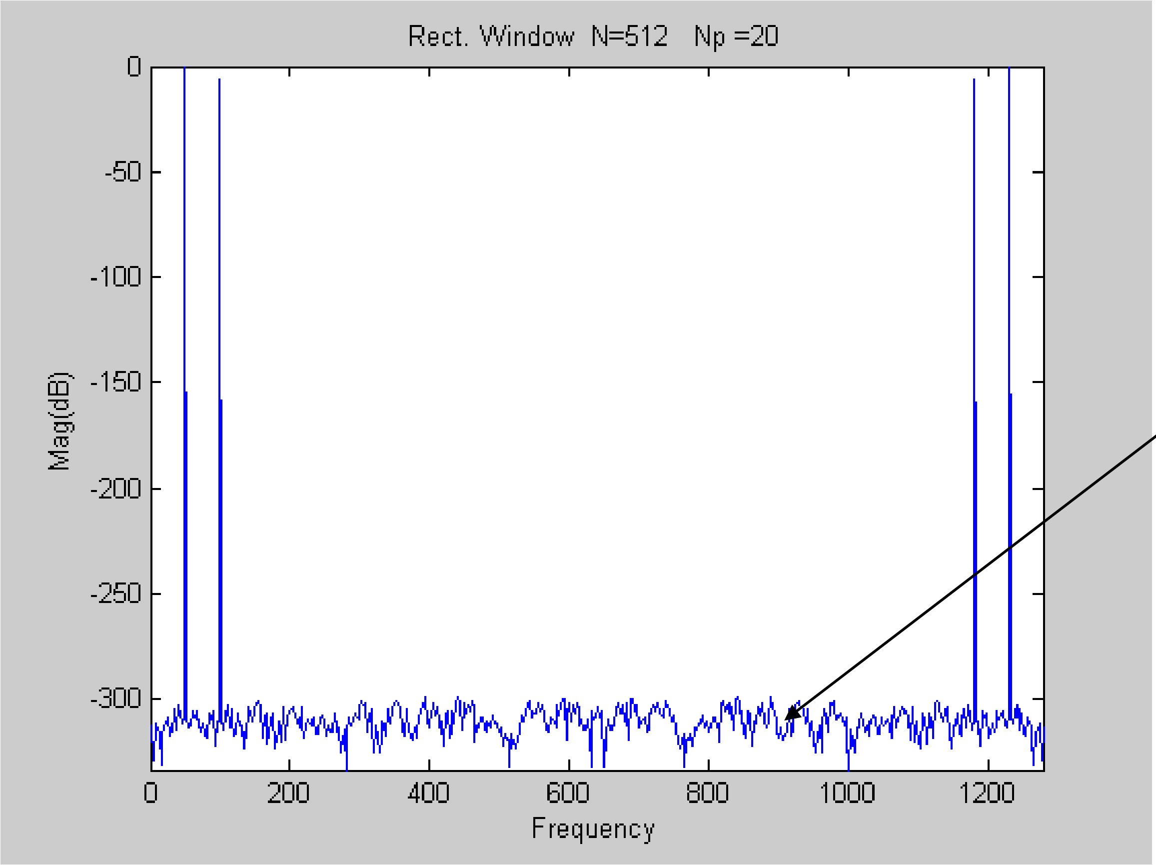
Columns 91 through 95

-309.8121 -311.6403 -312.8374 -310.5414 -308.7807

Columns 96 through 100

-316.7549 -316.3395 -308.4113 -307.3766 -311.0358

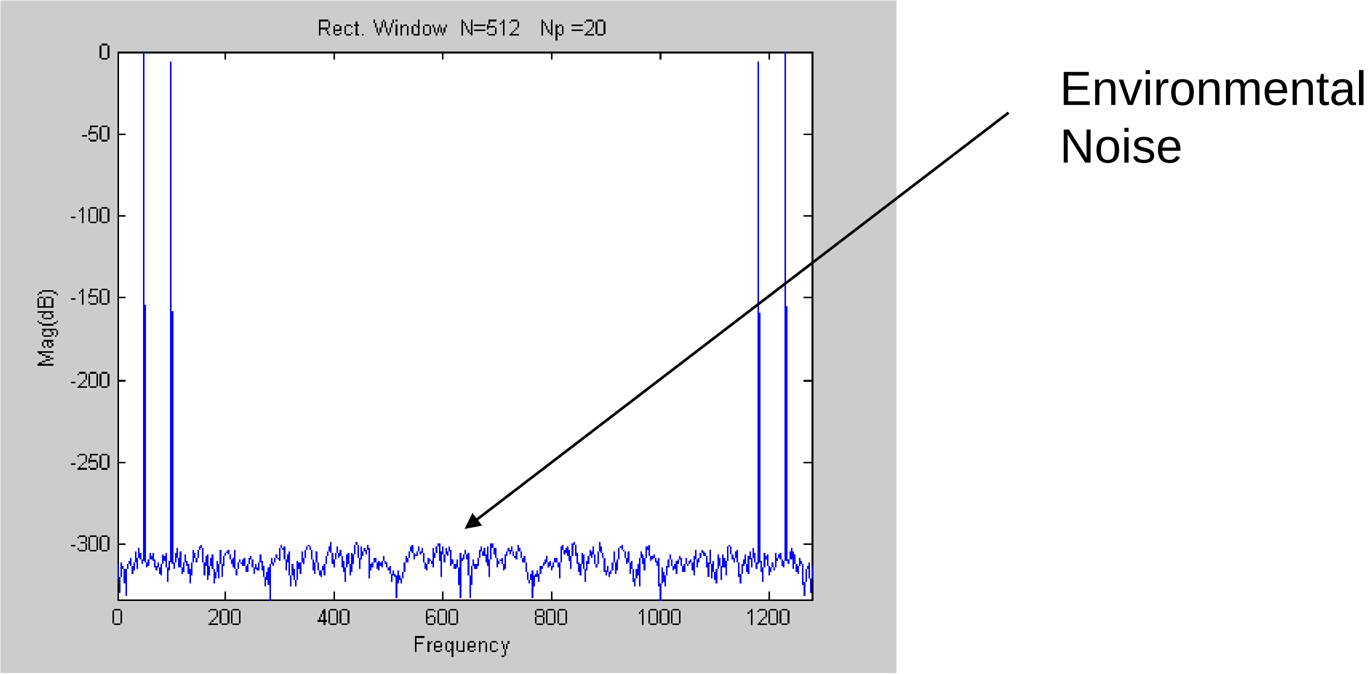
Question: How much noise is in the computational environment?



Environmental Noise

Is this due to quantization in the computational environment or to numerical rounding in the FFT?

Question: How much noise is in the computational environment?



Observation: This noise is nearly uniformly distributed
The level of this noise at each component is around -310dB

Question: How much noise is in the computational environment?

Assume A_k = -310 dB for $0 \le k \le N$

$$A_{kdB}=20\log10A_k \quad \Rightarrow \quad A_k = 10^{\frac{A_{kDB}}{20}}$$

$$A_k \cong 10^{\frac{-310}{20}} = 10^{-15.5} \overset{defn}{=} \bar{A}$$

$$V_{Noise,RMS} \cong \sqrt{\sum_{k=1}^{N-1}\left(\frac{A_k}{\sqrt{2}}\right)^2} \overset{\substack{A_k=\bar{A} \\ N\ large}}{=} \bar{A}\sqrt{\frac{N}{2}}$$

$$V_{Noise,RMS} \cong \bar{A}\sqrt{\frac{N}{2}} = 10^{-15.5}\sqrt{\frac{512}{2}} = 5.1 \bullet 10^{-15} \cong 5fV$$

This computational environment has a very low total computational noise and does not become significant until the 46-bit resolution level is reached !!

Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
- Importance of Satisfying Hypothesis
- Windowing

Example - Increase DFT length from 512 to 4096

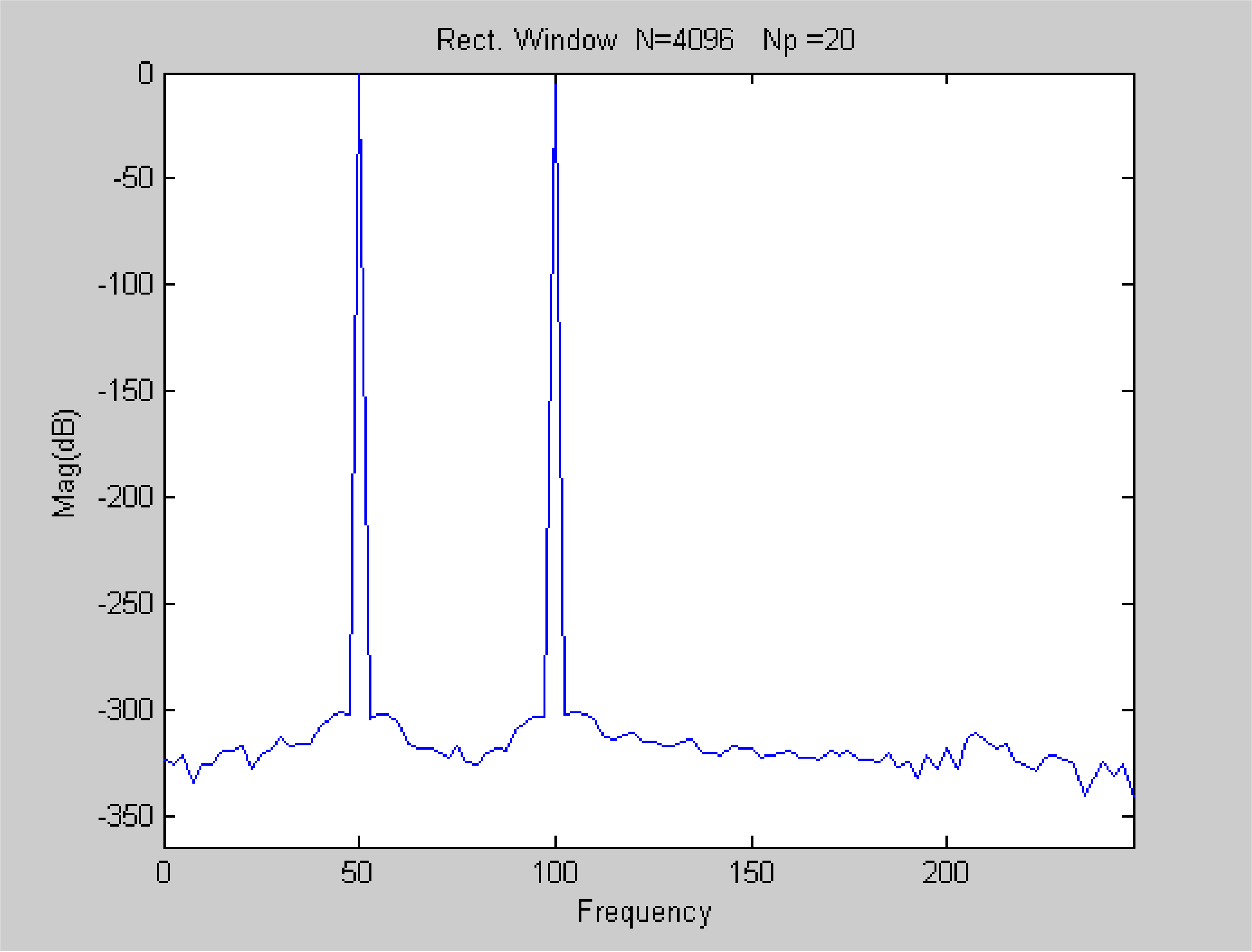
WLOG assume f_{SIG}=50Hz

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

Consider N_P=20 N=4096

Spectral Response



Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-323.9398 -325.5694 -321.3915 -334.6680 -325.2463 -325.3391 -319.3569

Columns 8 through 14

-319.7032 -317.4419 -327.4933 -321.1968 -318.2241 -312.7300 -316.8359

Columns 15 through 21

-315.5166 -316.1801 -307.8072 -304.3414 -301.3326 -301.7993 0

Columns 22 through 28

-303.9863 -302.2114 -302.5485 -306.5542 -315.4995 -318.3911 -318.4441

Columns 29 through 35

-318.7570 -322.6054 -317.3667 -324.0324 -325.8546 -320.3611 -317.8960

k^{th} harmonic will appear at position $1+k\bullet Np$

Columns 36 through 42

-319.0051 -309.4219 -305.5698 -302.8625 -303.2207 -6.0206 -302.3437

Columns 43 through 49

-300.8222 -301.6722 -304.8150 -313.0288 -313.5963 -312.1136 -310.7740

Columns 50 through 56

-314.7706 -315.3607 -317.0331 -316.8648 -314.4965 -314.3096 -320.4308

Columns 57 through 63

-320.2843 -320.9910 -316.8320 -318.3531 -318.4341 -322.1619 -321.6183

Columns 64 through 70

-320.6985 -319.0630 -322.1485 -322.3338 -323.6365 -319.0865 -321.0791

Example - Increase NP from 20 to 50

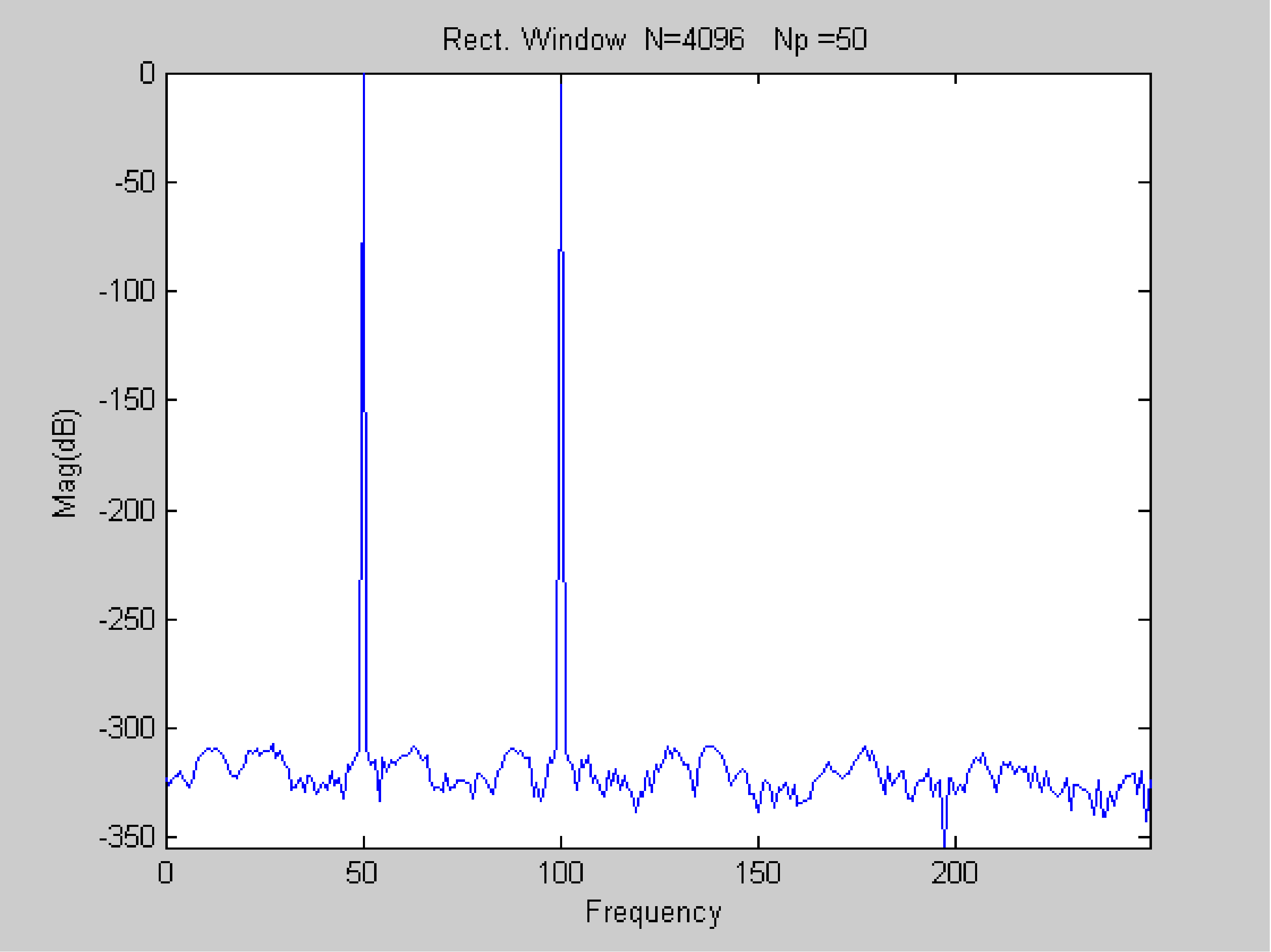
WLOG assume f_{SIG}=50Hz

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

Consider N_P=50 N=4096

Spectral Response



Fundamental will appear at position 1+Np = 51

Columns 1 through 7

-322.4309 -325.5445 -322.2645 -321.6226 -319.5894 -323.4895 -327.3216

Columns 8 through 14

-321.2981 -316.1855 -312.3071 -310.4889 -309.6790 -309.9436 -309.3734

Columns 15 through 21

-311.4435 -314.7665 -317.1248 -321.7733 -323.0602 -318.2119 -317.4601

Columns 22 through 28

-310.1735 -311.1633 -308.9079 -312.0709 -310.6683 -310.6908 -307.6761

Columns 29 through 35

-312.9440 -310.5706 -316.2098 -318.9565 -327.6885 -326.4021 -322.3135

Fundamental will appear at position 1+Np = 51

Columns 36 through 42

-328.5059 -321.5592 -322.6183 -330.2002 -328.5051 -324.3480 -328.0173

Columns 43 through 49

-319.3974 -325.8498 -323.1539 -331.9531 -317.0166 -318.3041 -314.9011

Columns 50 through 56

-309.5231 0 -308.8842 -316.1343 -314.5406 -333.4024 -313.7342

Columns 57 through 63

-319.6023 -314.9029 -316.6932 -314.7123 -311.9567 -312.0200 -309.8825

Columns 64 through 70

-308.7103 -309.8064 -314.9393 -312.4610 -322.7229 -328.0350 -326.6767

k^{th} harmonic will appear at position 1+k•Np

Columns 71 through 77

-329.1687 -321.1102 -328.3790 -326.9774 -323.4227 -323.3388 -325.1652

Columns 78 through 84

-325.3417 -332.1905 -320.4431 -322.1461 -323.8993 -325.4370 -329.8160

Columns 85 through 91

-319.1702 -317.1792 -312.4734 -310.2585 -309.5426 -310.8963 -310.6955

Columns 92 through 98

-313.6855 -313.3882 -330.4962 -324.4762 -333.2237 -325.8694 -313.9127

Columns 99 through 105

-315.4869 -308.6364 -6.0206 -309.2723 -314.4098 -316.3311 -328.2626

k^{th} harmonic will appear at position 1+k•Np

Columns 106 through 112

-314.3378 -317.7599 -312.1738 -324.4699 -321.7568 -326.3796 -331.0818

Columns 113 through 119

-319.9292 -325.4840 -318.0998 -328.0000 -321.7632 -326.5097 -328.5867

Columns 120 through 126

-338.0360 -328.6163 -330.5881 -319.7260 -329.2289 -316.3840 -319.1143

Columns 127 through 133

-315.0684 -308.6315 -312.9640 -309.5056 -311.6251 -316.1369 -316.1064

Columns 134 through 140

-320.4989 -331.2686 -314.3479 -310.0891 -308.0023 -308.1556 -309.0616

k^{th} harmonic will appear at position 1+k•Np

Columns 141 through 147

-311.2372 -312.6180 -319.0565 -325.6750 -323.7759 -320.7444 -318.0752

Columns 148 through 154

-320.5965 -330.3083 -330.2507 -338.2118 -325.0839 -323.5993 -326.2350

Columns 155 through 161

-336.0163 -326.5945 -327.9587 -324.7636 -332.5650 -326.1828 -334.9208

Columns 162 through 168

-333.9169 -333.3995 -332.0925 -324.3599 -322.9393 -320.4507 -317.7706

Columns 169 through 175

-315.9825 -319.2534 -320.8277 -322.3018 -321.6497 -320.4065 -315.4057

Considerations for Spectral Characterization

Quantization Noise

It will be shown that the quantization that takes place in either an ADC or a DAC acts like noise and is nearly uniformly distributed in all DFT bins.

Thus the deviations in output of data converters caused by magnitude quantization is termed quantization noise

It will be shown later that the RMS value of the quantization noise is given by the expression

$$E_{QUANT} \cong \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1}}$$

Quantization noise components in DFT bins are much larger than the computational noise which is also nearly uniformly distributed in all DFT bins

Considerations for Spectral Characterization

DFT Length and NP

- DFT Length and NP do not affect the computational noise floor

- Although not shown here yet, DFT length does reduce the quantization noise floor coefficients but not total quantization noise

Stay Safe and Stay Healthy !

End of Lecture 29